

Penguin B Decays $b \rightarrow sl^+l^-$ and $b \rightarrow sg$ *

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Abstract

The penguin mediated processes $b \rightarrow sg$ and $b \rightarrow sl^+l^-$ are studied. In the Standard Model, for the leptonic modes improvement in experimental limits will put stringent bounds on the top mass, where the present limit from $b \rightarrow s\mu^+\mu^-$ is 390 GeV. For hadronic penguin processes, although the gluonic penguin dominates, we find the electroweak contribution are around 30% for the upper range allowed top mass. The branching ratio for $B \rightarrow X_s\phi$ is predicted to be in the range $(0.6 \sim 2) \times 10^{-4}$. Effects of the charged Higgs in two Higgs doublet models are discussed.

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Rare B decays, particularly pure penguin decays, have been subject of considerable theoretical and experimental interest recently [1]. The photonic penguin induced process $B \rightarrow K^* \gamma$ has been observed by CLEO collaboration [2] and is consistent with the Standard Model (SM) prediction [3]. In this talk we will concentrate on two other classes of penguin decays, $b \rightarrow sl^+l^-$ and $b \rightarrow sg$.

Process $b \rightarrow sl^+l^-$

The process $b \rightarrow sl^+l^-$ is sensitive to top mass unlike $b \rightarrow s\gamma$, and improvement in the experimental bound should greatly improve the top quark mass upper limit which is at present at ~ 390 GeV from $b \rightarrow s\mu^+\mu^-$ [4]. This process for large top mass has dominant contribution from Z exchange and the box diagram [5].

The effective Hamiltonian density relevant for $b \rightarrow sl^+l^-$ decay is:

$$H_{eff} \cong \frac{4G_F}{\sqrt{2}} (V_{cb}V_{cs}^*) \sum \tilde{c}_j(m) \tilde{O}_j(m) . \quad (1)$$

The important operators for us are:

$$\begin{aligned} \tilde{O}_7 &= (e/16\pi^2) m_b (\bar{s}_L \sigma_{\mu\nu} b_R) F^{\mu\nu} , \\ \tilde{O}_9 &= (e^2/16\pi^2) (\bar{s}_L \gamma_\mu b_L) \bar{\ell} \gamma^\mu \ell , \\ \tilde{O}_{10} &= (e^2/16\pi^2) (\bar{s}_L \gamma_\mu b_L) \bar{\ell} \gamma^\mu \gamma_5 \ell . \end{aligned} \quad (2)$$

Here $F_{\mu\nu}$ is the electromagnetic interaction field strength tensor.

The QCD-renormalized coefficients $\tilde{c}_j(m)$ are calculated in Ref. [6], and their implications are discussed in Ref. [4]. The branching ratio of $b \rightarrow sl^+l^-$ can be written after normalizing the rate to $BR(b \rightarrow ce\bar{\nu}) \approx 0.108$, [6,7]:

$$BR(b \rightarrow sl^+l^-) = K [F_1(|\tilde{c}_9|^2 + |\tilde{c}_{10}|^2) + F_3\tilde{c}_9\tilde{c}_7 + F_2|\tilde{c}_7|^2] , \quad (3)$$

where

$$K = (\alpha/4\pi)^2 (2/\lambda\tilde{\rho}) BR(b \rightarrow ce\bar{\nu}) = 1.6 \cdot 10^{-7} \quad (4)$$

and α is fine structure constant. The phase space factor $\tilde{\rho}$ and the QCD correction factor λ for the semileptonic process are well known [8]. We have used $\tilde{\rho} = 0.5$ and $\lambda = 0.889$. The phase space integration from $min = (2m_\ell/m_b)^2$ to $max = (1 - m_s/m_b)^2$ give the following values [9] for the constants F_i :

$$F_1 = 1, \quad F_3 = 8, \quad \text{for } min \cong 0, \quad max \cong 1 , \quad (5)$$

$$F_2 = 32 [\ln(m_b/2m_\ell)] ; \quad \text{for } max \cong 1; \quad \ell = e, \mu . \quad (6)$$

We plot in Fig. 1 the branching ratios for $b \rightarrow sl^+l^-$ as a function of the top mass for the standard model. In the SM the process $b \rightarrow se^+e^-$ is enhanced over $b \rightarrow s\mu^+\mu^-$ by $\sim 60\%$

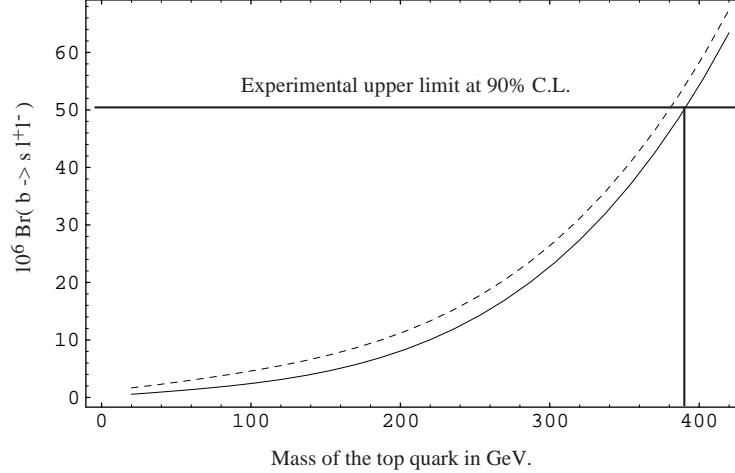


FIG. 1. Branching ratios for $b \rightarrow s e^+ e^-$ (dashed line) and $b \rightarrow s \mu^+ \mu^-$ (solid line) as a function of the top mass.

for $m_t = 150$ GeV due to the small electron mass [9]. As noted, the decay rate is highly dependent on m_t and improvement on this limit should improve bounds on m_t significantly.

In Ref. [4] the implications of additional Higgs doublet on $b \rightarrow s l^+ l^-$ is discussed. The conclusions are:

- 1) In two-Higgs doublet model II, where couplings are like the minimal supersymmetric model, constraints are possible for all values of $\tan \beta = v_2/v_1$ for smaller Higgs masses; although the experimental bounds will have to be improved to draw useful conclusions.
- 2) In the two-Higgs doublet model I, the constraints on Higgs masses are similar to $b \rightarrow s \gamma$, and tight constraints can be derived on Higgs masses only for small values of $\tan \beta = v_2/v_1$.

Process $b \rightarrow sg$

The gluonic penguin induced B decays are expected to be observed very soon. A large number of gluonic penguin induced B decay channels were studied in Ref. [10] using $\Delta B = 1$ effective Hamiltonian $H_{\Delta B=1}$ in the lowest nonvanishing order. In Ref. [11] the next-to-leading order QCD corrected pure gluonic penguin $H_{\Delta B=1}$ was used with top quark mass m_t fixed at 150 GeV. In this talk we present a study of the next-to-leading order QCD corrected Hamiltonian $H_{\Delta B=1}$ in the SM and in two Higgs doublet models, taking particular care to *include the full electroweak contributions* and find the dependence on m_t and α_s . The cleanest signature of hadronic penguin processes are: $B \rightarrow X_s \phi$, $B \rightarrow K \phi (K^* \phi)$, and $B_s \rightarrow \phi \phi$. The process $B \rightarrow X_s \phi$ is particularly recommended because it is free from form factor uncertainties. We find not only that the QCD correction in next-to-leading order are large, but also inclusion of the full electroweak contributions have significant effect on the branching ratio which could reduce the pure gluonic penguin contribution by 30% at the upper range of allowed top quark mass. Our results which have been derived independently [12], agree with Fleischer [13]. The electroweak corrections alter the isospin structure of penguins, and have a major impact on the analysis of certain B decays. This will be presented

$m_t(\text{GeV})$	c_1	c_2	c_3	c_4	c_5
130	-0.313	1.150	0.017	-0.037	0.010
174	-0.313	1.150	0.017	-0.037	0.010
210	-0.313	1.150	0.018	-0.038	0.010
$m_t(\text{GeV})$	c_6	c_7/α_{em}	c_8/α_{em}	c_9/α_{em}	c_{10}/α_{em}
130	-0.045	-0.061	0.029	-0.978	0.191
174	-0.046	-0.001	0.049	-1.321	0.267
210	-0.046	0.060	0.069	-1.626	0.334

TABLE I. The Wilson coefficients for $\Delta B = 1$ at $m_b = 5 \text{ GeV}$ with $\alpha_s(m_Z) = 0.118$.

in a forthcoming publication [14].

The QCD corrected $H_{\Delta B=1}$ relevant to us can be written as follows [15]:

$$H_{\Delta B=1} = \frac{G_F}{\sqrt{2}} [V_{ub}V_{us}^*(c_1O_1^u + c_2O_2^u) + V_{cb}V_{cs}^*(c_1O_1^c + c_2O_2^c) - V_{tb}V_{ts}^* \sum c_i O_i] + H.C. , \quad (7)$$

where the Wilson coefficients (WCs) c_i are defined at the scale of $\mu \approx m_b$; and O_i are defined as

$$\begin{aligned} O_1^q &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) q_\beta \bar{q}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha , \\ O_2^q &= \bar{s} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma^\mu (1 - \gamma_5) b , \\ O_3 &= \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{q}' \gamma_\mu (1 - \gamma_5) q' , \\ Q_4 &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \bar{q}'_\beta \gamma_\mu (1 - \gamma_5) q'_\alpha , \\ O_5 &= \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{q}' \gamma^\mu (1 + \gamma_5) q' , \\ Q_6 &= \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \bar{q}'_\beta \gamma_\mu (1 + \gamma_5) q'_\alpha , \\ O_7 &= \frac{3}{2} \bar{s} \gamma_\mu (1 - \gamma_5) b e_{q'} \bar{q}' \gamma^\mu (1 + \gamma_5) q' , \\ Q_8 &= \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta e_{q'} \bar{q}'_\beta \gamma_\mu (1 + \gamma_5) q'_\alpha , \\ O_9 &= \frac{3}{2} \bar{s} \gamma_\mu (1 - \gamma_5) b e_{q'} \bar{q}' \gamma^\mu (1 - \gamma_5) q' , \\ Q_{10} &= \frac{3}{2} \bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta e_{q'} \bar{q}'_\beta \gamma_\mu (1 - \gamma_5) q'_\alpha . \end{aligned} \quad (8)$$

Here q' is summed over u, d, and s.

We work with renormalization scheme independent WCs c_i as discussed in Ref. [15]. In Table 1, we show some sample values of c_i for some values of m_t with the central value $\alpha_s(m_Z) = 0.118$ and $\mu = m_b$ [12].

We also need to treat the matrix elements to one-loop level for consistency. These one-loop matrix elements can be rewritten in terms of the tree-level matrix elements $\langle O_j \rangle^t$ of the effective operators, and one finds [13,16] $\langle c_i Q_i \rangle$ to be equal to

$$c_i [\delta_{ij} + \frac{\alpha_s}{4\pi} m_{ij}^s + \frac{\alpha_{em}}{4\pi} m_{ij}^e] \langle O_j \rangle^t \equiv c_i^{eff} \langle O_i \rangle^t . \quad (9)$$

We have worked out the full matrices $m^{s,e}$. For the processes we are considering only c_{3-10} contribute. These are given by,

$$\begin{aligned} c_3^{eff} &= c_3 - P_s/3, & c_4^{eff} &= c_4 + P_s, \\ c_5^{eff} &= c_5 - P_s/3, & c_6^{eff} &= c_6 + P_s, \\ c_7^{eff} &= c_7 + P_e, & c_8^{eff} &= c_8, \\ c_9^{eff} &= c_9 + P_e, & c_{10}^{eff} &= c_{10}. \end{aligned} \quad (10)$$

The leading contributions to $P_{s,e}$ are given by: $P_s = (\alpha_s/8\pi)c_2(10/9 + G(m_c, \mu, q^2))$ and $P_e = (\alpha_{em}/9\pi)(3c_1 + c_2)(10/9 + G(m_c, \mu, q^2))$. Here m_c is the charm quark mass which we take to be 1.35 GeV. The function $G(m, \mu, q^2)$ is give by

$$G(m, \mu, q^2) = 4 \int_0^1 x(1-x) dx \ln \frac{m^2 - x(1-x)q^2}{\mu^2}. \quad (11)$$

In the numerical calculation, we will use $q^2 = m_b^2/2$ which represents the average value.

We obtain the decay amplitude for $B \rightarrow X_s \phi$

$$\begin{aligned} A(B \rightarrow X_s \phi) &\approx A(b \rightarrow s \phi) = \\ &= -\frac{g_\phi G_F}{\sqrt{2}} V_{tb} V_{ts}^* \epsilon^\mu C \bar{s} \gamma_\mu (1 - \gamma_5) b, \end{aligned} \quad (12)$$

where ϵ^μ is the polarization of the ϕ particle; $C = c_3^{eff} + c_4^{eff} + c_5^{eff} + \xi(c_3^{eff} + c_4^{eff} + c_6^{eff}) - (c_7^{eff} + c_9^{eff} + c_{10}^{eff} + \xi(c_8^{eff} + c_9^{eff} + c_{10}^{eff}))/2$ with $\xi = 1/N_c$, where N_c is the number of colors. The coupling constant g_ϕ is defined by $\langle \phi | \bar{s} \gamma^\mu s | 0 \rangle = i g_\phi \epsilon^\mu$. From the experimental value for $Br(\phi \rightarrow e^+ e^-)$, we obtain $g_\phi^2 = 0.0586 \text{ GeV}^4$.

The decay rate is, then, given by

$$\begin{aligned} \Gamma(B \rightarrow X_s \phi) &= \frac{G_F^2 g_\phi^2 m_b^3}{16\pi m_\phi^2} |V_{tb} V_{ts}^*|^2 |C|^2 \lambda_{s\phi}^{3/2} \\ &\times [1 + \frac{3}{\lambda_{s\phi}} \frac{m_\phi^2}{m_b^2} (1 - \frac{m_\phi^2}{m_b^2} + \frac{m_s^2}{m_b^2})], \end{aligned} \quad (13)$$

where $\lambda_{ij} = (1 - m_j^2/m_b^2 - m_i^2/m_b^2)^2 - 4m_i^2 m_j^2/m_b^4$.

We normalize the branching ratio to the semi-leptonic decay of $B \rightarrow X_c e \bar{\nu}_e$. We have

$$\begin{aligned} Br(B \rightarrow X_s \phi) &= Br(B \rightarrow X_c e \bar{\nu}_e) \frac{|V_{tb} V_{ts}^*|^2}{|V_{cb}|^2} \\ &\times \frac{12\pi^2 g_\phi^2 \lambda_{s\phi}^{3/2}}{m_\phi^2 m_b^2 \lambda_{\bar{\nu}}^{3/2}} |C|^2 [1 + \frac{3}{\lambda_{s\phi}} \frac{m_\phi^2}{m_b^2} (1 - \frac{m_\phi^2}{m_b^2} + \frac{m_s^2}{m_b^2})]. \end{aligned} \quad (14)$$

We show in, Figure 2 and 3 the predictions for the branching ratio $Br(B \rightarrow X_s \phi)$ in the SM as a function of top quark mass m_t and the strong coupling constant $\alpha_s(m_Z)$ with and without electroweak corrections, and for $N_c = 2$ and 3.

The dominant contribuitons are from the gluonic penguin. There is a very small m_t dependence for the branching ratio calculated without the inclusion of the electroweak penguin contributions. The inclusion of the full electroweak contribuitons have sizeable effects which reduce the branching ratios by about 20% to 30% for the central value of α_s with m_t varying

from 100 GeV to 200 GeV. It is clear from Figure 2 and 3 that the full contribution has a large m_t dependence. There may be corrections to the branching ratios predicted by the factorization method. It is a common practice to parameterize the possible new contributions by treating ξ as a free parameter [17–19]. Using experimental values from non-leptonic B decays, it is found that [18], $a_1 = c_2 + \xi c_1$ and $a_2 = c_1 + \xi c_2$ have the same signs, and $|a_2| \approx 0.27$ and $|a_1| \approx 1.0$. The branching ratios for $N_c = 2$ are about 2 times those for $N_c = 3$.

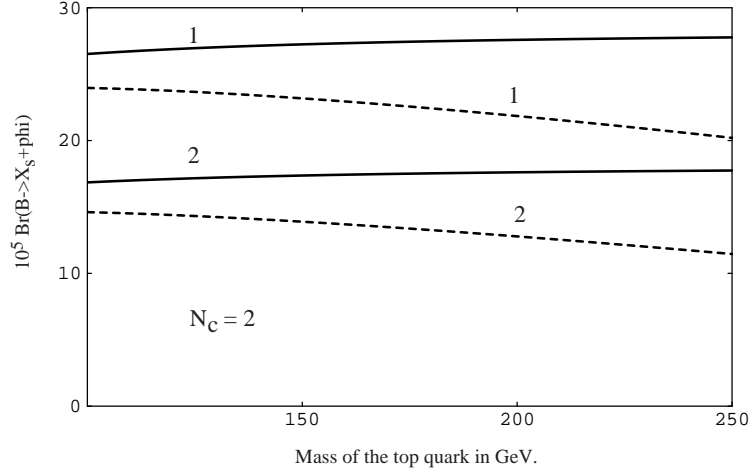


FIG. 2. $Br(B \rightarrow X_s \phi)$ as a function of top mass with $N_c = 2$, and $\alpha_s(m_Z) = 0.125$ (curves 1) and $\alpha_s(m_Z) = 0.111$ (curves 2). The dashed and solid lines are for the branching ratios with the full strong and electroweak penguin contributions, and without the electroweak contributions, respectively.

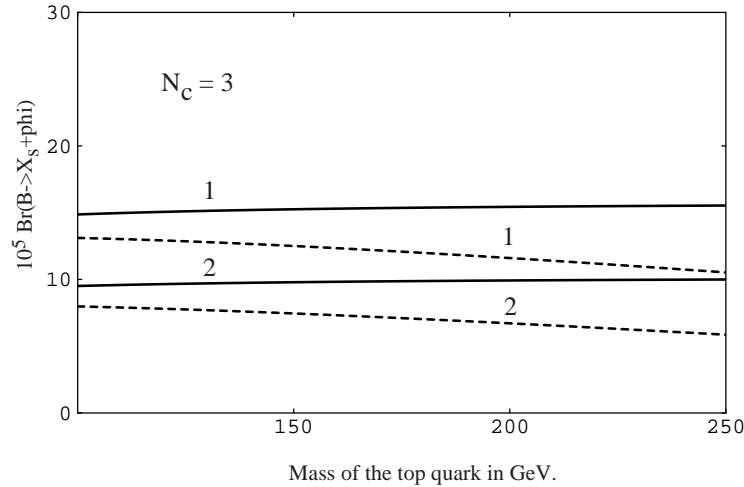


FIG. 3. The same as FIG.2 but with $N_c = 3$.

For the central value of $\alpha_s(m_Z)$ and the central value of $m_t = 174$ GeV reported by CDF [20], the value for $Br(B \rightarrow X_s \phi)$ is about 1.7×10^{-4} for $N_c = 2$.

Using form factors from Refs. [17,18], we also calculated the exclusive decay rates for $B \rightarrow K \phi$, $B \rightarrow K^* \phi$, and $B_s \rightarrow \phi \phi$. The exclusive branching ratios $B \rightarrow K \phi$ and $B \rightarrow K^* \phi$

are about the same which are 1×10^{-5} if the form factors from Ref. [17] are used. If the form factors from Ref. [18] are used, one obtains $Br(B \rightarrow K\phi) \approx 1.7 \times 10^{-5}$, $Br(B \rightarrow K^*\phi) \approx 0.5 \times 10^{-5}$, and $Br(B_s \rightarrow \phi\phi) \approx 0.4 \times 10^{-5}$.

We have also looked at $b \rightarrow sg$ in two-Higgs doublet model [12]. Both models I and II give the same results. The ratio of decay rates of the SM predictions to the two-Higgs doublet model predictions weakly depends on N_c . We find that the effects of the charged Higgs boson contributions are small for $\cot\beta < 1$. When increasing $\cot\beta$, the charged Higgs contributions become important and the effect is to cancel the SM contributions. When $\cot\beta$ becomes very large the charged Higgs boson contributions become the dominant ones. However, using the information from $B \rightarrow X_s\gamma$, it is found that for small $m_H \sim 100$ GeV and $m_t \sim 174$ GeV, $\cot\beta$ is constrained to be less than 1 [21]. For these values, the charged Higgs boson effects on the processes discussed in this paper are less than 10%. For $m_H \sim 500$ GeV, the charged Higgs boson effects can reduce the hadronic penguin B decays by 40% because the range of $\cot\beta$ allowed from $b \rightarrow s\gamma$ is now larger [21]. The effects become smaller for larger m_H .

$B \rightarrow K\pi/\pi\pi$ modes

We now present a summary of contribution by Hayashi, Joshi, Matsuda and Tanimoto [22] to this conference. They focus on gluonic effects in the exclusive channels $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$. These channels have contributions from both the tree operators $O_{1,2}$ and the penguin operators. their Hamiltonian includes leading order QCD correction, but not the Z, γ penguin and W box contributions to the penguin diagrams. Effect of charm loop is included in the same manor as discussed by us where c_i^{eff} are introduced. The value of q^2 is taken as $m_b^2/2$ in P_s . The factorization hypothesis is employed, with further assumption that $N_c = 3$, which might lead to incorrect estimates.

The amplitude for $B^0 \rightarrow K^+\pi^-$ and $B^0 \rightarrow \pi^+\pi^-$ can both be expressed in terms of a universal form factor $F_0^{B\pi}(q^2)$ if B annihilation terms are neglected

$$q^\mu < \pi^- | \bar{u}\gamma_\mu b | B_d^0 > = (m_B^2 - m_\pi^2) F_0^{B\pi}(q^2). \quad (15)$$

This form factor drops out when ratios of $B^0 \rightarrow K^+\pi^-$ to $B^0 \rightarrow \pi^+\pi^-$ decay rates are taken. This ratio is, however, sensitive to $|V_{ub}/V_{cb}|$ and the phase of $V_{ub} = |V_{ub}|e^{-i\phi}$. The authors find $R_B = \Gamma(B_b^0 \rightarrow K^-\pi^+)/\Gamma(B_d^0 \rightarrow \pi^+\pi^-)$ can range from 0.4 to 7.0. They also calculated the relative contribution of penguin and tree contributions to $B \rightarrow K\pi$ and $B \rightarrow \pi\pi$ processes. Their results for ratio of amplitudes for $\phi = 90^\circ$ are

$$\frac{A(penguin)}{A(tree)} = \begin{cases} 4.22 \frac{0.08}{|V_{ub}/V_{cb}|}, & \text{for } B \rightarrow K\pi \\ 0.22 \frac{0.08}{|V_{ub}/V_{cb}|}, & \text{for } B \rightarrow \pi\pi \end{cases} \quad (16)$$

The present CLEO observation of $BR(B \rightarrow K^+\pi^- + \pi^+\pi^-) = (2.4_{-0.7}^{+0.8} \pm 0.2) \times 10^{-5}$ imposes the limit $F_0^{B\pi}(0) = 0.26$ to 0.55 which is consistent with BSW model $F_0^{B\pi}(0) = 0.33$.

The authors have also considered CP asymmetry in $B \rightarrow \pi^+\pi^-$ decay arising from the phase of V_{ub} as well as the imaginary part c^{eff} . The asymmetry can be as large as 30%.

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